Study on height growth model of *Pinus koraiensis*

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Abstract: Growth model is an efficient way to study growing process of some factors of plants quantitatively. Height growth of Korean pine (*Pinus koraiensis*) was studied by using Hyperbola equation, Logistic equation, Richards equation with three parameters, and Richards equation with four parameters in this paper. The results showed that Richards equation with four parameters was the most suitable and could be turned into other theoretical equations when some parameters were given different value. The maximum height of trees could be given in advance when using Richards equation with four parameters, and it was even more corresponding to reality. In addition, a height growth model with real height of fixed age as a parameter was discussed in this paper. This kind of growth model could be used to calculate height growth of a given tree effectively.

Key words: Korean pine; Height growth; Model

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Introduction

Growth model is an efficient way to study growing process of trees quantitatively, and it is the theoretical basis for managing measure of forests and can be used to evaluate and forecast growth of trees. Growth model includes individual tree growth model and stand growth model chiefly, and the individual tree growth model is the basis for studies on stand growth model.

Korean pine (*Pinus koraiensis*) is one of the valuable tree species and the main afforestation tree species in Northeast China. Some studies have been made on individual tree growth model of Korean pine (Jiang 1985; Deng *et al.* 1997; 1999), which aimed at simulating height growth formula for Korean pine and working out site index table, and these had an important signification to guide forestry practice. Taking Korean pine as example, this research emphasized farther studies and discussion on height growth model.

Materials and methods

The data of stem analysis of Korean pine were provided by Academy of Forestry Sciences of Jilin Province. The data were investigated in Yichun Area and Changbai Mountain, where are the chief distributing area of Korean pine.

Height growth model includes experiential model and theoretical model (Richards 1959; Cooper 1961; Turnbull 1963; Jiang 1985; Deng *et al.* 1997; 1999), and the equations in common use are as follows:

Hyperbola equation $$H = \frac{A^2}{aA^2 + b} \tag{1}$$

Logistic equation $$H = \frac{K}{1 + ae^{-bA}} \tag{2}$$

Mitscherlich equation $$H = K(1 - ae^{-bA}) \tag{3}$$

Richards equation with 3 parameters

$$H = K(1 - e^{-bA})^c \tag{4}$$

Richards equation with 4 parameters

$$H = K(1 - ae^{-bA})^c \tag{5}$$

In above equations, H stands for height of tree, A stands for age, and K, a, b, and c are different parameters. Equation (2), (3), and (4) are all theoretical models, and they can be transformed from equation $W^{1-m} = K^{1-m} - Ae^{-bt}$ while some parameters are given different value. The equation (5) can be transformed from a modified Logistic equation (Deng *et al.* 1997; 1999), and it also can be transformed into Equation (2), (3), and (4) (Deng *et al.* 1999). In general, parameter K stands for the maximum height that trees can reach, and b stands for growth speed. Parameter a and c have not clear biological meaning, but they can affect flexuosity and position of growth curve.

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Hyperbola equation, Logistic equation, and Richards equation were used to simulate the growth of Korean pine because of their preferable effect (Richards 1959; Deng *et al.* 1997; 1999). Although Mitscherlich equation is one kind of theoretical models, it simulates only the growth of quick growing trees well (Deng *et al.* 1997; 1999). The original data were modified by equation $H_i = h_i + \frac{h_{i+1} - h_i}{2(r_i - r_{i+1})}$ at first (Wang 1992; Deng *et al.* 1997), and then the growth of Korean pine was simulated by using SYSTAT (Wilkinson 1990) and the mentioned equations.

Results

The growth models of Korean pine were simulated as the following equation (Fig.1), and in these equations, the number of samples (n) was 21.

$$H = \frac{A^2}{0.033A^2 + 275.734} \tag{6}$$

(SSe=10.761, Corrected R^2=0.991)

$$H = \frac{26.958}{1 + 8.96e^{-0.025A}} \tag{7}$$

(SSe=3.469, Corrected R^2=0.997)

$$H = 32.368(1 - e^{-0.01A})^{1.461} \tag{8}$$

(SSe=1.032, Corrected R^2=0.999)

$$H = 28.71(1 - 0.096e^{-0.017A})^{32.258} \tag{9}$$

(SSe=0.245, Corrected R^2=1.000)

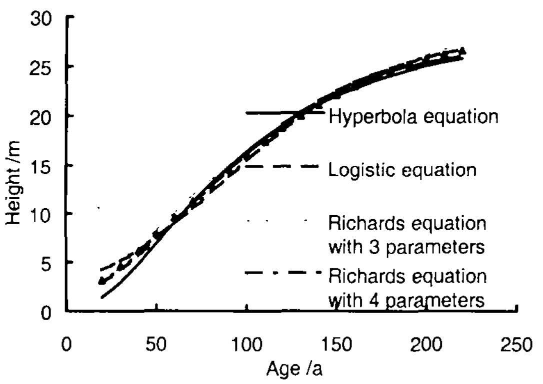

Fig.1 Height growth curves of Korean pine by four kinds of equations

According to SSe and corrected R^2 (Wilkinson 1990), all the equations were simulated well, and Richards equation with 4 parameters (equation (9)) was the best. As one kind of experiential model, Hyperbola equation could simulate height growth well too, and its parameters could also have biological meaning in certain extent (Deng *et al.* 1997).

In equation (7) and (8), values of K were 26.958 and 32.368 respectively, and they expressed the maximum heights from these two equations. There existed obvious difference between the values of K, and it showed that different growth models could cause different results. To resolve this problem, equation (5) was a good selection. Because equation (5) had 4 parameters, an appropriate value of K could be given in advance, and the result could be perfect. To discuss this point, three different values of K (26.958, 30, and 32.368) were given to simulate height growth as follows:

$$H = 26.958(1 + 2.613e^{-0.022A})^{-2.027} \tag{10}$$

(SSe=2.2, Corrected R^2=0.998)

$$H = 30(1 + 0.005e^{-0.015A})^{-643485} \tag{11}$$

(SSe=0.977, Corrected R^2=0.999)

$$H = 32.368(1 + 0.00141e^{-0.013A})^{-1915.34} \tag{12}$$

(SSe=6.050, Corrected R^2=0.995)

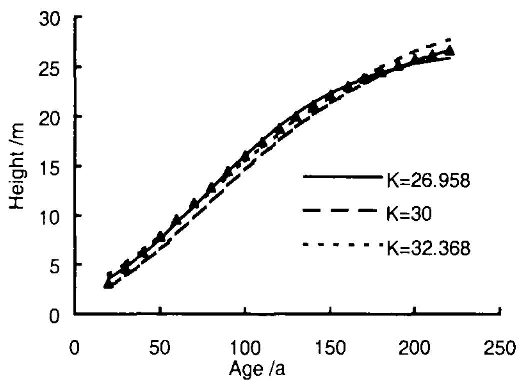

Fig.2 Height growth curves of Richards equations with 4 parameters and different K

Obviously, all the equations with different values of K simulated well. Furthermore, the more close to 28.71 the value was, the more perfect the simulating was (Fig.2). In fact, height growth of trees was related to site condition closely, as simulating height growth by using Richards equation with 4 parameters, a suitable value of K according to site condition could be given in advance. For the Mitscherlich equation, Logistic equation, and Richards equation with 3 parameters, because of their less parameters, their adaptabilities were not as good as that of equation (5), and they could not be used under giving a certain value of K.

To simulate height growth by using equation (5), although actual site condition and the potential maximum height were considered, the process of height growth was not accordant in the same site because of some reasons. So, selecting height of certain age as a parameter in the growth equation and establishing this equation were necessary. Therefore, the original stem analysis data of Ko-

rean pine were transacted into corresponding data with ages from 20 a to 220 a at an interval of 10 a, and then, values of $\ln(h_{i+1})$ to values of $\ln(h_i)$ were plotted (Fig. 3). From Fig. 3, there existed good linear relation between $\ln(h_{i+1})$ and $\ln(h_i)$, and the linear equation was expressed as equation (13).

$$\ln(h_{i+1}) = 0.834\ln(h_i) + 0.55 \quad (13)$$

(SSe=0.002, Adjusted R^2=1.000, p<0.000)

In equation (13), $\ln(h_{i+1})$ means logarithmic value of height with ages from 30 a to 220 a at an interval of 10 a, and $\ln(h_i)$ means logarithmic value of height with ages from 20 a to 210 a at an interval of 10 a. Equation (13) could be transformed to equation (14).

$$h_{i+1} = e^{0.55} h_i^{0.834} \quad (14)$$

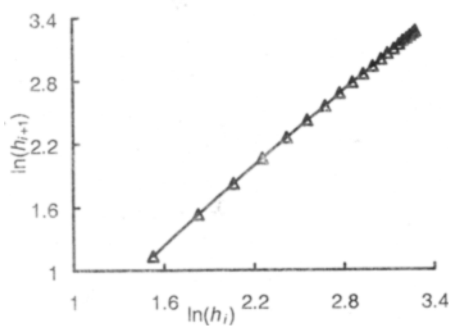

Fig.3 Relationship between $\ln(h_{i+1})$ and $\ln(h_i)$

So, selecting height at age of 20 as criterion and a determinate parameter, height of age at an interval of 10 a could be expressed as equation (15), and heights of $i*10$ a could be calculated.

$$h_{i*10} = e^{\frac{0.55(1-0.834^{i-2})}{0.166}} h_{20}^{(0.834^{i-2})} \quad (15)$$

Generally, in the S-typed curve which was often used to reflect growth of biology, there existed linear relation between $\frac{1}{W_{i+1}}$ and $\frac{1}{W_i}$ or between $\ln(\frac{1}{W_{i+1}})$ and $\ln(\frac{1}{W_i})$. W stands for the existing quantity of biological growth. So, equation (15) had determinate theoretical basis (Deng *et al* 1999).

Conclusions

Growth model is an efficient way to study growing process of trees quantitatively. It includes experiential model and theoretical model, and the equations in common use are Hyperbola equation, Mitscherlich equation, Logistic equation, and Richards equation. They can be used to evaluate and forecast growth of trees, and it is the theoretical basis for managing measure of forests. Although Hyperbola equation simulated the growth of Korean pine well in this research, most of experimental equations have limitations, and the results from experimental equations have not good biological meaning. Mitscherlich equation, Logistic equation, and Richards equation are theoretical equations in common use, and they simulate growth well. Although Mitscherlich equation is one kind of theoretical models, it simulates only the growth of quickly growing trees well. Richards equation, as a flexible curve, has a wide application.

Richards equation with four parameters is one kind of Richards equation, and it can be transformed from a modified Logistic equation, furthermore, it also can be transformed into Mitscherlich equation, Logistic equation, and Richards equation with three parameters. While simulating height growth by using Richards equation with 4 parameters, a suitable value of K according to site condition could be given in advance, and it can get a better simulation.

Generally, in the S-typed curve which was often used to reflect growth of biology, there existed linear relation between $\frac{1}{W_{i+1}}$ and $\frac{1}{W_i}$ or between $\ln(\frac{1}{W_{i+1}})$ and $\ln(\frac{1}{W_i})$.

So, equation $h_{i*10} = e^{\frac{0.55(1-0.834^{i-2})}{0.166}} h_{20}^{(0.834^{i-2})}$ had determinate theoretical basis and simulated height growth well.

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